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Hydrogen Molecules and the Hayashi Effect [and Discussion]

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Hydrogen molecules and the Hayashi effect

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The modifications to the Hayashi effect due to a high-opacity atmosphere or the presence of a molecular hydrogen dissociation zone are investigated. The importance of pressure dissociation in the latter case demands first an investigation of this problem. It is concluded that neither effect modifies the Hayashi effect very greatly. Some speculation on the effect of variability on the Hayashi effect is advanced.

1. Introduction

Hayashi (1961) has shown that no quasi-static stellar models can exist with effective temperatures less than some critical value. Since this temperature is in the region of 3000 °K it is of considerable importance if astronomers working in the infrared are to discover very cool sources.

If p and t are variables describing pressure and temperature in the envelope of a star, made dimensionless with suitable combinations of the mass and radius of the star, then if the envelope of the star is convective:

$$pt^{-\frac{5}{2}} = E \text{ (a constant)}.$$

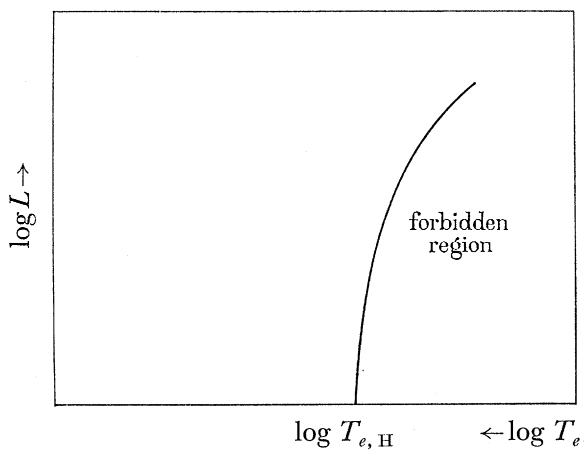

Figure 1. General character of the forbidden region for given mass and composition.

Hayashi showed that for a fully convective star $E = 45$, and that including a radiative zone in the star decreases E; so that no interior solution to the stellar structure equations exists if $E > 45$. By a study of the surface region for each mass and composition he was able to relate the condition $E > 45$ with a region of the H–R diagram—the 'forbidden' region. He found the position of the top of the convective zone was the most important factor in deciding where the forbidden region should be, and that this in turn was fixed by the position of the hydrogen ionization zone. Figure 1 shows the general character of the forbidden region obtained by Hayashi to be a region $T_e < T_{e,\mathrm{H}}$. $T_{e,\mathrm{H}}$ is a critical temperature that has only a weak dependence on mass, luminosity and composition.

There are two effects not considered by Hayashi that might *a priori* be expected to modify $T_{e,\mathrm{H}}$: (i) the existence of a high-opacity atmosphere resulting from the presence of carbon grains, (ii) the presence of a hydrogen molecular dissociation region.

The effects of (i) should be quite small since Hayashi, Hoshi & Sugimoto (1962) have shown that:

$$T_{e,\mathrm{H}} \propto \kappa^{-\frac{1}{13}}$$

where κ is the opacity; however, an increase in opacity should make the forbidden region smaller. On the other hand one would intuitively expect effect (ii) to increase $T_{e,\mathrm{H}}$ and the size of the forbidden region, since the destabilization in the H_2 dissociation region might also lead to convection.

2. Method of investigation

The method adopted to study the effects (i) and (ii) was to integrate the stellar structure equations inwards from the top of the star's atmosphere to a temperature of $T = 10^5$ °K—this region being defined as the envelope. This is essentially half a stellar structure computation. The mass in the envelope is computed and compared with the total mass of the star: if it exceeds the total mass then the initial conditions chosen for the top of the atmosphere lie in the forbidden region.

It is important for the investigation of this problem that a theory of convection be chosen as free from arbitrariness as possible. Therefore the theory of Faulkner, Griffith & Hoyle (1965) (F.G.H.) was chosen since it avoided the arbitariness inherent in mixing-length theories. In these theories the position of the forbidden region is sensitive to the value of the mixing length. In the F.G.H. theory of convection it is argued that only the falling convective elements contribute to the convective flux since they become more isolated from their surroundings whereas rising elements are hotter and more transparent than their surroundings. Since this is so it is possible to integrate equations for $\langle v^2 \rangle$, the mean square velocity, and $\langle \Delta T \rangle$, the mean temperature defect of the falling elements, and so to establish the point at which the convection becomes efficient at carrying the flux from the star.

This technique will underestimate $T_{e,\mathrm{H}}$, since if the envelope were chosen to extend to a temperature of say 10^6 °K, some initial conditions that exhaust their mass before reaching this depth, may not have done so before reaching 10^5 °K. However, one may reasonably hope that this approach will illustrate the importance of effects (i) and (ii).

The results of effect (i) are simply investigated by setting

$$\kappa = 30 \quad \text{if} \quad T < 2500 \text{ °K},$$

the latter temperature being approximately that at which carbon grains vaporize.

The investigation of effect (ii) is complicated by the problem of pressure dissociation.

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3. Pressure dissociation

Vardya (1960) first pointed out the importance of pressure dissociation of molecules in late-type stars. If one takes the normal expression for the dissociation constant:

$$K_p = T_4^{\frac{3}{4}} \exp\left(29\cdot439 - \frac{D}{T_4}\right) \times \left(1 - \exp\left(-\frac{0\cdot6155}{T_4}\right)\right), \qquad (1)$$

where T_4 is the temperature measured in units of 10^4 °K and D is the dissociation potential measured in the same units (in fact $D = 5\cdot182$) then this expression is independent of P. In a red giant star, owing to the fact that P increases more rapidly than T going into the star,

$$X_2 \sim \tfrac{1}{2}X \quad \text{at} \quad T = 10^4\ °\text{K},$$

where X_2 and X are the proportions by mass of hydrogen molecules and all hydrogen. This is obviously unrealistic since ionization is already occurring at these temperatures.

Vardya (1965) states that pressure dissociation arises due to exchange forces between atoms and molecules and is not primarily due to the obliteration of the upper energy levels. It occurs when the inter-particle separation is of the same order as the separation of atoms in a molecule.

It should be noted that the formulation of pressure dissociation presented here is different from that of Vardya and those differences will be highlighted in what follows. What one requires from a theory of pressure dissociation is (*a*) a means of calculating X_2, (*b*) a formula for the binding energy of the medium—this fact is needed to be able to compute the adiabats.

Let us assume that every two hydrogen atoms, whether in molecules or not, attract each other with a force deduced from the familiar Rydberg potential:

$$U(r) = -7\cdot605_{10} - 12(1+2\cdot04\xi)\exp(-2\cdot04\xi), \qquad (2)$$

where $\xi = r/r_e - 1$, and r_e is the equilibrium separation of the atoms in a hydrogen molecule. Here we differ from Vardya who actually solved the three-body problem for the inter-particle forces when one is bound into the molecule. His results differ essentially by only a factor 1·2 from (2).

If the mean density of hydrogen atoms (whether bound or not) is n_{H}, and if we assume this is independent of position (unaffected by the forces resulting from (2)), we have that the mean potential due to all hydrogen atoms in the system is

$$\overline{U} = \int_0^\infty 4\pi r^2 n_{\text{H}} U(r)\,\mathrm{d}r = n_{\text{H}} \int_0^\infty 4\pi r^2 U(r)\,\mathrm{d}r. \qquad (3)$$

We now define the 'depression of the continuum'

$$\Delta D = \overline{U} \propto n_{\text{H}} \propto \rho. \qquad (4)$$

Now figure 2 shows the interpretation of this result. First, the potential experienced by one member of a molecule due to the presence of its companion is shown as the curve *XCEY*. Then the potential due to all the other atoms in the system is the line *ACB*. Whereas without pressure dissociation a molecule to dissociate had to have energy D, now if it has only energy D–ΔD this will be sufficient to free it from its companion since it can reach C

and become subject to the field of all the other atoms. Here the status of the escaped particle is similar to that of a particle in a solution since while it has escaped from its companion, it is still bound into the medium as a whole.

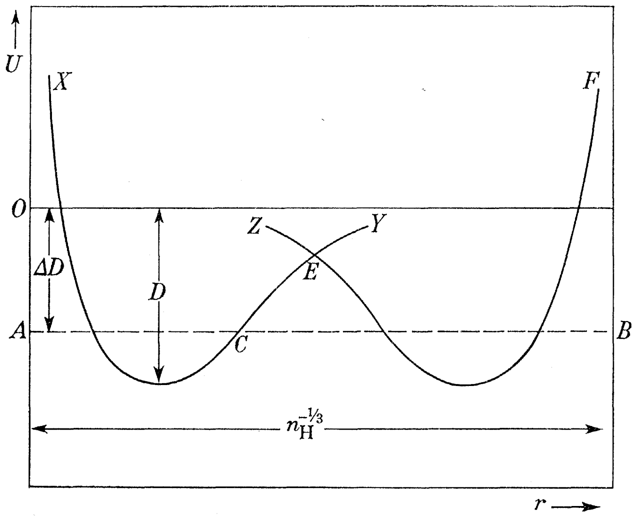

FIGURE 2. Interpretation of pressure dissociation scheme. For explanation see text.

On the other hand, Vardya's scheme is to place the third particle in his system at the mean inter-particle separation $1/n_H^{\frac{1}{3}}$. The potential of this particle in figure 2 is represented by the curve *FEZ* and Vardya uses the point *E* in place of *C* in our theory. It is felt that the scheme presented here is an improvement on that of Vardya since particles slightly closer than the equilibrium distance will have greater effects than those slightly further apart, and that it is necessary to perform some averaging procedure such as (3). Moreover, our equation (4) is a much simpler result than that of Vardya.

So we meet requirement (*a*) by replacing *D* by $D - \Delta D$ in (1). It remains to specify the binding energy of the medium, which is needed both for the adiabats and for the thermal capacity of the convective elements. We write

$$\text{binding energy} = Dn_2 + \tfrac{1}{2}\Delta D n_{\text{I}},$$

where n_2 and n_{I} are the numbers of hydrogen molecules and free neutral hydrogen atoms respectively. Note that as well as the contribution from the binding energy of the molecules, there is also a contribution due to interaction of neutral atoms through chemical forces. The factor $\frac{1}{2}$ is due to the fact we are examining a two-particle interaction and ionized atoms are taken not to experience exchange-type forces.

4. Results and discussion

A full account of the detailed equations integrated, together with the results of a survey of the forbidden region for various masses and compositions, will be published elsewhere. The computations were performed on the ICT 1909 computer in the Nautical Almanac Office and checked against F.G.H.s solar model. Here the results for stars of solar mass, luminosity and composition are presented in figure 3. The results show that the critical temperature unmodified by effects (i) and (ii) is *ca.* 2500 °K. Including grain opacity gives $T_{e,\mathrm{H}} \sim 2000$ °K, while including molecular dissociation gives $T_{e,\mathrm{H}} \sim 2800$ °K. Both effects combined gives the same results as the latter, since the grain opacity does not make possible any extra models cooler than $T_e = 2500$ °K and the models with $T_e > 2500$ °K are unaffected by the grain opacity. So the results do not show any unexpected features—in fact $T_{e,\mathrm{H}}$ is somewhat increased.

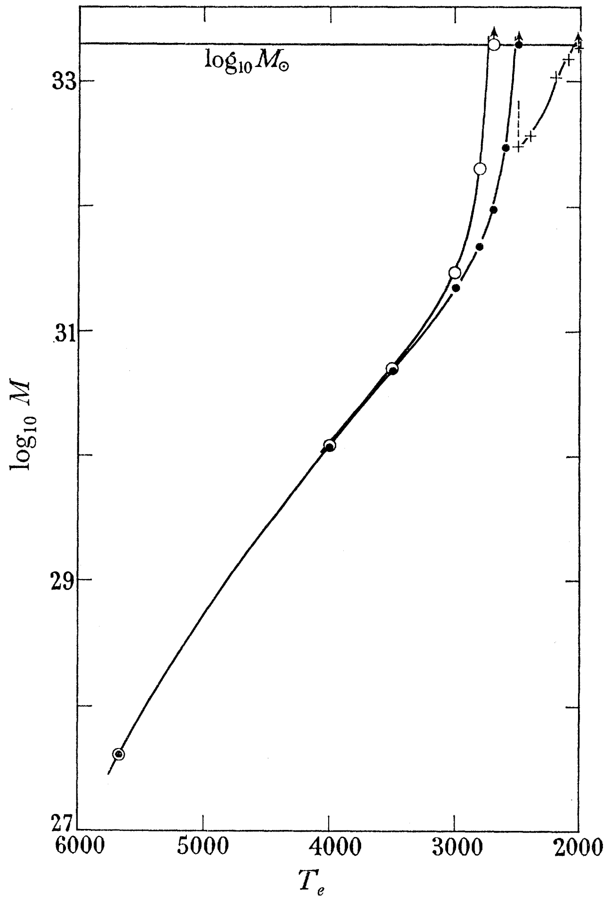

FIGURE 3. Plot of mass in the envelope against effective temperature for stars of solar mass, luminosity and composition. ●, Normal opacity; +, grain opacity; ○, normal opacity but with molecular dissociation.

On the other hand, observations show that there are stars with $T_e > T_{e,\mathrm{H}}$. O. J. Eggen (1967, private communication), using narrow-band photometry that avoids the blanketing due to molecular bands in late-type stars, has examined many red stars and found that all stars redder than a certain value are variable. This is not in conflict with our results, which refer only to quasi-static stars. The only exception to this known to the author is the star NML Cygnus, but various authors (Poveda 1965; Penston 1966; Low & Smith 1966) have suggested that in proto-stars the atmosphere is supported by rotation, so that again the above analysis may not be applicable.

However, the fact that variable stars may lie in the forbidden region suggests that effect (iii)—variability—may be important in this context. Here the difficulty arises that the interaction of convection and mass-motion is poorly understood. However, one may speculate as follows: no convection is possible during free-fall since there is no gravitation against which to convect. If the atmosphere of late-type variable stars are in free-fall during most of the cycle in the same way as many other variables then convection can only arise when the motion of the atmosphere is reversed. Also this reversal may occur very suddenly, for example by the action of a shock wave, so that this period may be too brief to allow convection to become established. Then the top of the convection zone is driven deeper into the star and $T_{\epsilon,\mathrm{H}}$ may be lowered for variable stars.

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References (Penston)

Faulkner, J., Griffith, K. & Hoyle, F. 1965 *Mon. Not. R. Astr. Soc.* **129**, 363.
Hayashi, C. 1961 *Publ. Astr. Soc. Japan* **13**, 450.
Hayashi, C., Hoshi, R. & Sugimoto, D. 1962 *Prog. Theor. Phys. Suppl.* **22**, 83.
Low, F. J. & Smith, B. J. 1966 *Nature, Lond.* **212**, 675.
Penston, M. V. 1966 *Obsy.* **86**, 121.
Poveda, A. 1965 *Boll. Ton. y Tac.* **26**, 15.
Vardya, M. S. 1960 *Astrophys. J.* **132**, 905.
Vardya, M. S. 1965 *Mon. Not. R. Astr. Soc.* **129**, 345.

Discussion

H. Bondi, F.R.S.

Professor Bondi reminded the meeting of the relevance to the present discussion of some points in a short paper by Gold, Hoyle and himself (*Observatory* 1955). That paper pointed out that with almost any plausible concentration of hydrogen molecules there would be very substantial amounts of infrared radiation from clouds of inter-stellar matter at the temperature of 100 °K, which had then been strongly suggested by observations. The paper then approached the question of the origin of this energy which, given the amounts, could hardly be other than nuclear. Thus such a radiating cloud, with a surface temperature of around 100 °K, might be thought of as a Black Giant star, though of course the shape might be most irregular and the location of the nuclear energy generation not particularly regularly shaped either. Such Black Giant stars might help to maintain the irregular motions of matter within the galaxy.

Other events have overtaken some of the speculations of the paper, but modern discoveries of infrared sources again suggest something on the lines of Black Giant stars.